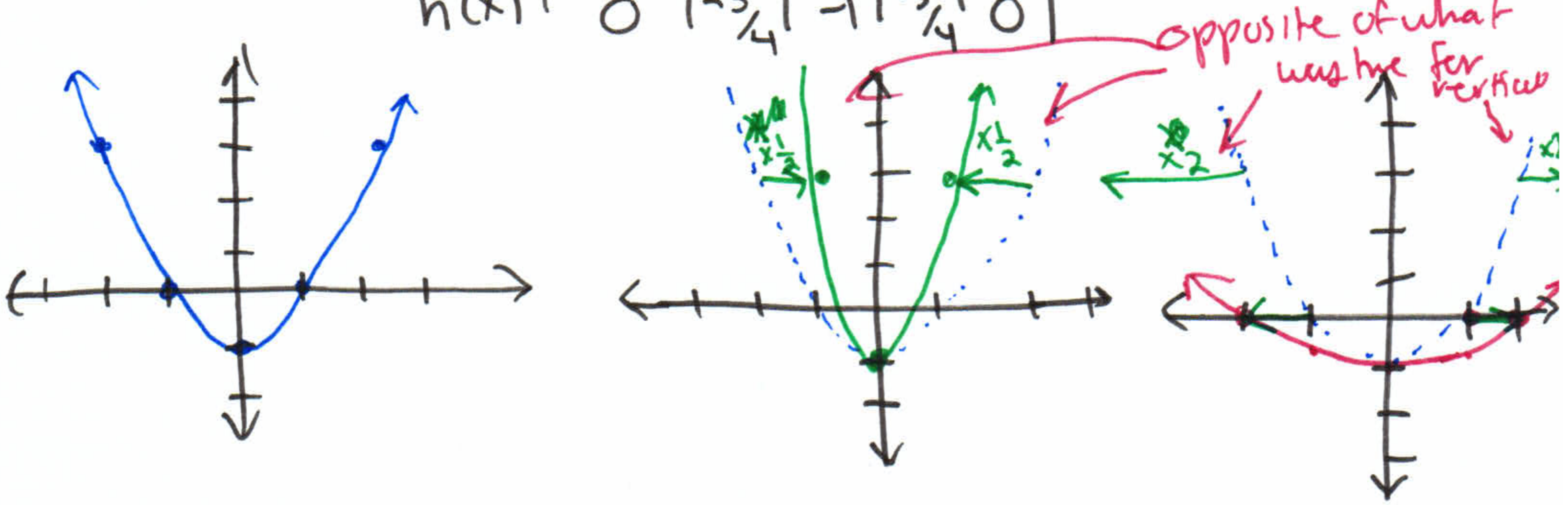
Lecture 10/30/2023: Horizontal Stretches and compressions

Quiz 8 on Friday

$f(x) = x^2 - 1$; $g(x) = f(2x) = (2x)^2 - 1$; $h(x) = f(\frac{1}{2}x) = (\frac{1}{2}x)^2 - 1$

x	-2	-1	0	1	2
$f(x)$	3	0	-1	0	3
$g(x)$	15	3	-1	3	15
$h(x)$	0	$-3/4$	-1	$-3/4$	0



Defn: Let $f(x)$ be a function and $k \neq 0$. Then, the graph of $y = f(a \cdot x)$ is

1) Compressed horizontally by a factor of $|a|$ if $|a| > 1$
2) Stretched horizontally by a factor of $\frac{1}{|a|}$ if $0 < |a| < 1$
3) Reflected about y-axis if $a = -1$.

Rmk: Vertical Stretches + Compressions don't change x-int.

Horizontal compressions and stretches don't change y-int.

<u>Ex</u>: $f(x) = x^2 + 2$. Write an explicit formula for each function and describe its graph.

a) $6f(x) = 6(x^2 + 2)$

$f(x)$ stretched vert. by a factor of 6

b) $\frac{1}{3}f(3x) = \frac{1}{3}\left((3x)^2 + 2\right)$

$f(x)$ ~~stretched vert.~~ compressed horiz. by a factor of 3

and then vertically compressed by a factor of 3

<u>Note</u>: ~~Doing vertica~~ We can choose to do vertical translations first or horizontal. ~~When~~ But we have to be careful when we have multiple vertical transl.

c) $f(4x) = (4x)^2 + 2$

f compressed horiz. by a factor of 4

d) $f(\frac{1}{2}x) = (\frac{1}{2}x)^2 + 2$

f stretched horiz. by a factor of 2.

<u>Ex</u>: Suppose Domain of $j(x)$ is $0 \le x \le 6$ and range is $-3 \le j(x) \le 3$. Determine the domain and range for:

a) $j(\frac{1}{4}x)$

divide ~~multiply~~ by factor

D: $0 \le x \le 4 \cdot 6 = 0 \le x \le 24$

R: $-3 \le f(\frac{1}{4}x) \le 3$

Range stays the same

b) $\frac{1}{3}j(2x)$

D: $0 \div 2 \le x$

a) $j(\frac{1}{4}x)$

3

Domain: We know j is well defined if $0 \leq x \leq 6$ so ~~then~~ in order for $0 \leq \frac{1}{4}x \leq 6$

we need $\boxed{0 \leq x \leq 24}$ ← ~~multiply~~ Divide by $\frac{1}{4}$

Range: Horizontal trans. don't change range

$R: \ -3 \leq j(\frac{1}{4}x) \leq 3.$

b) $\frac{1}{3}j(2x)$

Domain: We know j is well defined for $0 \leq x \leq 6$, so in order for

$$0 \leq 2x \leq 6$$

$$D: \boxed{0 \leq x \leq 3}$$ Divide by 2

Range: We multiply range of $j(x)$ by $\frac{1}{3}$

$R = -1 \leq \frac{1}{3}j(2x) \leq 1$ multiply by $\frac{1}{3}$